V-Formation as Optimal Control

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Abstract. We present a new formulation of the V-formation problem for migrating birds in terms of model predictive control (MPC). In our approach, to drive a collection of N birds towards a desired formation, an optimal *velocity adjustment* (acceleration) is performed at each time-step on each bird's current velocity using a model-based prediction window of T time-steps. We present both centralized and distributed versions of this approach. The optimization criteria we consider are based on fitness metrics of candidate accelerations that V-formations are known to exhibit. These include *velocity matching*, *clear view*, and *upwash benefit*. We validate our MPC-based approach by showing that for a significant majority of simulation runs, the flock succeeds in forming the desired formation. Our results help to better understand the emergent behavior of formation flight, and provide a control strategy for flocks of autonomous aerial vehicles.

1 Introduction

It has long been observed that flocks of birds organize themselves into V-formations, particularly migrating birds traveling long distances. There are two main reasons for this behavior. The first relates to the aerodynamics of formation-flight, where birds generate an upwash region off the trailing edge of their wings, allowing birds behind them to save energy from this free lift [2, 11]. The second reason is that a V-formation provides birds with an optimum combination of a clear visual field along with visibility of lateral neighbors [5, 6].

Previous work on modeling this emergent behavior has focused on providing combinations of *dynamical flight rules* as driving forces. For example, in [4], the authors extend Reynolds's model [9] with a rule that forces a bird to move laterally away from any bird that blocks its view. This can result in multiple V-shaped clusters, but flock-wide convergence is not guaranteed. The work of [3] induces V-formations by extending Reynolds's model with a *drag reduction* rule, but the final formation tends to oscillate as birds repeatedly adjust the angle of attack. Another approach, based on three *positioning rules*, is that of [8]. Their model, however, is limited by the assumption that birds have a constant longitudinal heading. The authors of [10] attemtp to improve this approach by handling *turning movements*. But their model also produces small clusters of birds, each of which is only moderately V-like.

We, in contrast, view the problem of V-formation as one of *optimal control*. Compared to previous work, there are no behavioral rule in our approach. Instead, we adopt the idea of model predictive control (MPC) [1]. To drive a collection of N birds towards a desired formation, an optimal *velocity adjustment* (acceleration) is performed at each time-step on each bird's current velocity using a model-based prediction window of T time-steps. This approach yields an optimal acceleration sequence of length T, and the first acceleration in the sequence is applied. We present both centralized and distributed versions of this approach.

The optimization we perform is based on fitness metrics that capture the essence of a V-formation, namely *Velocity Matching* (VM), *Clear View* (CV), and *Upwash Benefit* (UB). VM means that bird velocities are aligned, allowing them to maintain formation. CV requires birds to have an unobstructed view, while UB models the energy saving birds obtain from the upwash regions generated by their frontal neighbors. We show by simulations that birds succeed in forming the desired formations with high probability.

2 Model Predictive Control for V-Formation

Let $\boldsymbol{x}(t)_i$, $\boldsymbol{v}(t)_i$ and $\boldsymbol{a}(t)_i$ be the vector of 2-dimensional positions, velocities and accelerations, respectively, of bird i at time t, $1 \leq i \leq N$. The following equations model the behaviors of bird i in discrete time:

$$\boldsymbol{x}(t+1)_i = \boldsymbol{x}(t)_i + \boldsymbol{v}(t+1)_i$$
$$\boldsymbol{v}(t+1)_i = \boldsymbol{v}(t)_i + \boldsymbol{a}(t)_i$$

Our MPC approach uses an optimization function to find the best acceleration $\boldsymbol{a}(t)_i$ at each time-step. Each bird optimizes its own acceleration based on local information about its nearest N_R neighboring birds. It tries to find the best accelerations of all of its neighbors including itself, and uses its own component of the solution to update its velocity and position. Let $\boldsymbol{x}_{N_i}$, $\boldsymbol{v}_{N_i}$ and $\boldsymbol{a}_{N_i}$ be the vector of positions, velocities and accelerations of bird i's neighbors. We consider the following optimization problem for bird i at time t:

$$\boldsymbol{a}^*_{N_i}(t), ..., \boldsymbol{a}^*_{N_i}(t+T-1) = \underset{\boldsymbol{a}_{N_i}(t),...,\boldsymbol{a}_{N_i}(t+T-1)}{\arg\min} J(\boldsymbol{a}_{N_i}(t+T-1), \boldsymbol{x}_{N_i}(t+T-1), \boldsymbol{v}_{N_i}(t+T-1))$$
$$\text{subject to} \quad \boldsymbol{x}_{N_i}(t), \boldsymbol{v}_{N_i}(t) = \texttt{Neighbors}(i, \boldsymbol{x}(t), \boldsymbol{v}(t), N_R);$$
$$\forall_{\tau \in [t, t+T-1]}, \boldsymbol{x}_{N_i}(\tau+1) = \boldsymbol{x}_{N_i}(\tau) + \boldsymbol{v}_{N_i}(\tau+1), \boldsymbol{v}_{N_i}(\tau+1) = \boldsymbol{v}_{N_i}(\tau) + \boldsymbol{a}_{N_i}(\tau);$$
$$\forall_{i \leq N_R} \ ||\boldsymbol{v}_{N_i}(\tau)_i|| \leq \boldsymbol{v}_{max}, ||\boldsymbol{a}_{N_i}(\tau)_i|| \leq \delta ||\boldsymbol{v}_{N_i}(\tau)_i||, \delta \in (0, 1).$$

where T is the prediction horizon, and J is the fitness function. Function `Neighbors` returns the positions and velocities of the nearest N_R birds of bird i (including i) at time t. We place a constraint on the maximum velocities and accelerations. We apply $\boldsymbol{a}(t)_i = \boldsymbol{a}^*_{N_i}(t)_i$ as the optimal acceleration for bird i at time t.

We also consider a *centralized approach* in which birds have information about the entire flock, i.e. $N_R = N$. In this case, we only need to perform one optimization for all birds at each time-step. The fitness function J consists of a sum-of-squares combination of VM, CV and UB. Let $\boldsymbol{v}' = \boldsymbol{v}(t) + \boldsymbol{a}(t)$ and $\boldsymbol{x}' = \boldsymbol{x}(t) + \boldsymbol{v}'$ be the new velocities and positions after applying the accelerations,

$$J(\boldsymbol{a}(t), \boldsymbol{x}(t), \boldsymbol{v}(t)) = (VM(\boldsymbol{v}') - VM^*)^2 + (CV(\boldsymbol{x}', \boldsymbol{v}') - CV^*)^2 + (UB(\boldsymbol{x}', \boldsymbol{v}') - UB^*)^2$$

where $VM^* = 0, CV^* = 0, UB^* = 1$ are the optimal values in a V-formation.

3 Fitness Metrics

Velocity Matching. The velocity matching metric is defined as $VM(\boldsymbol{v}) = \sum_{i>j} \left(\frac{||v_i - v_j||}{||v_i|| + ||v_j||} \right)^2$ where v_i is bird i's velocity. The optimal value in a V-formation is $VM^* = 0$, as all birds will have the same velocity, enabling them to maintain formation.

Clear View. The clear visual field is a cone with angle θ that can be blocked by the wings of other birds. We define the clear-view metric by accumulating the percentage of a bird's visual field that is blocked by other birds:

$$B_{ij}(h_{ij}, v_{ij}) = \begin{cases} \{\alpha \mid \max(\frac{\pi - \theta}{2}, \text{atan}(\frac{v_{ij}}{h_{ij} + w})) \leq \alpha \leq \min(\frac{\pi + \theta}{2}, \text{atan}(\frac{v_{ij}}{h_{ij} - w}))\} \\ \qquad \text{if } (h_{ij} < w \vee \frac{h_{ij} - w}{v_{ij}} < \tan\theta) \wedge \texttt{Front}(j, i); \\ \emptyset \qquad \text{otherwise.} \end{cases}$$

$$CV_i(\boldsymbol{x}, \boldsymbol{v}) = \frac{|\bigcup_{j \neq i} B_{ij}(h_{ij}, v_{ij})|}{\theta}, \quad CV(\boldsymbol{x}, \boldsymbol{v}) = \sum_i CV_i(\boldsymbol{x}, \boldsymbol{v})$$

where w is the wing span of a bird, $|S|$ is the size of a set S, h_{ij} and v_{ij} is the horizontal and vertical distance between i and j w.r.t. the direction of i's velocity, respectively, which can be computed using $\boldsymbol{x}$ and $\boldsymbol{v}$. Function $B_{ij}(h_{ij}, v_{ij})$ computes the range of i's view angle being blocked by j, and $CV_i(\boldsymbol{x}, \boldsymbol{v})$ computes the percentage of i's view that is blocked by other birds. Predicate $\texttt{Front}(j, i)$ is true when bird j is in front of bird i. The optimal value in a V-formation $CV^* = 0$, as all birds have the clear visual field.

Upwash Benefit. Upwash is generated near the wingtips of a bird, while downwash is generated near the center of a bird. We accumulate all birds' upwash benefits using a Gaussian-like model of the upwash and downwash region. The upwash and downwash a trailing bird i obtains from a preceding bird j is given by:

$$UB_{ij}(h_{ij}, v_{ij}) = \begin{cases} \frac{v_i \cdot v_j}{||v_i|| \cdot ||v_j||} S(h_{ij}) \cdot G(h_{ij}, v_{ij}, \mu_1, \Sigma_1) & \text{if } h_{ij} \geq \frac{(4-\pi)w}{8} \wedge \texttt{Front}(j, i) \\ S(h_{ij}) \cdot G(h_{ij}, v_{ij}, \mu_2, \Sigma_2) & \text{if } h_{ij} < \frac{(4-\pi)w}{8} \wedge \texttt{Front}(j, i) \\ 0 & \text{otherwise} \end{cases}$$

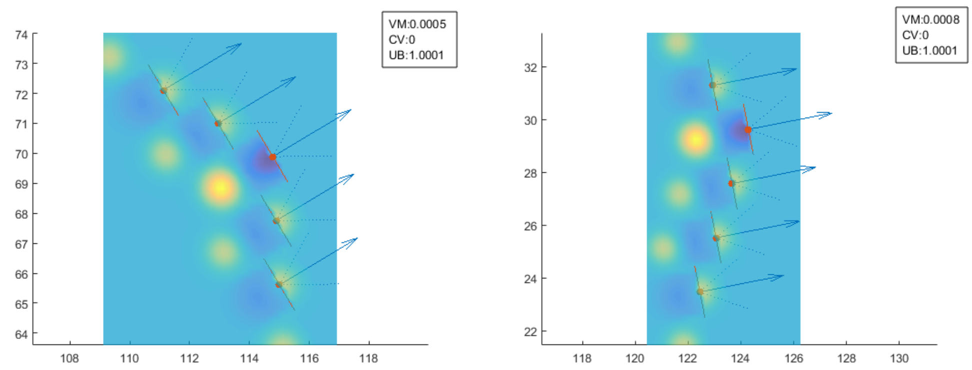
$$S(h_{ij}) = \texttt{erf}(2\sqrt{2}(h_{ij} - \tfrac{(4-\pi)w}{8})), \quad G(h_{ij}, v_{ij}, \mu, \Sigma) = e^{(-\frac{1}{2}([h_{ij}, v_{ij}] - \mu)^T \Sigma^{-1}([h_{ij}, v_{ij}] - \mu))}$$

where w is the wing span, $h_{ij} = (4 - \pi)w/8$ is the boundary between upwash and downwash region [7], $S(h_{ij})$ is a smoothing function with $\texttt{erf}$ being the error function, and $G(h_{ij}, v_{ij}, \mu, \Sigma)$ is a Gaussian-like function. Parameters μ_1, μ_2 are chosen such that the upwash benefit is maximized when $h_{ij} = (12+\pi)w/16$ [7] and $v_{ij} = 1$, and minimized when $h_{ij} = 0$ and $v_{ij} = 0$. Moreover, bird i only gets maximum upwash if the velocities of i and j are aligned; so the upwash is discounted by $\frac{v_i \cdot v_j}{||v_i|| \cdot ||v_j||}$. The total upwash benefit of the whole flock is $UB(\boldsymbol{x}, \boldsymbol{v}) = \sum_i (1 - min(\sum_j UB_{ij}(h_{ij}, v_{ij}), 1))$. The maximum upwash a bird can obtain is constrained to not be greater than 1. The optimal value in a V-formation $UB^* = 1$, as there is one leader that does not get any upwash.

4 Experimental Results

We used MATLAB function `particleswarm` from the Global Optimization Toolbox as the optimization algorithm. We placed a collision-avoidance constraint on the minimum distance between any two birds. The optimizer discards accelerations that will lead to collisions. The initial positions and velocities are randomly chosen with maximum velocity $\boldsymbol{v}_{max} = 5$. The bound on acceleration $\delta_{model} = 0.5$ for the model and $\delta_{plant} = 0.4$ for the plant. If the acceleration that the model produces exceeds the limit of the plant, we keep its direction and use the plant upper bound for its magnitude. We ran simulations of 50 time-steps with prediction horizon $T = 1$.

Fig. 1 shows the formations reached in the last step of simulation. We ran five simulation starting from random initial conditions for both centralized and distributed control. In both cases, four of the five simulations resulted in on-target formations. Future work will focus on further improving the success rate.



(a) Centralized Control (b) Distributed Control. $N_R = 3$

Fig. 1: Final formations from simulations with 5 birds. The red-filled circle and two protruding line segments represent a bird's body and wings with wing span $w = 1$. Arrows represent bird velocities. Dotted lines illustrate clear-view cones with angle $\theta = \pi/3$. A brighter background color indicates a higher upwash, while a darker background color indicates a higher downwash.

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